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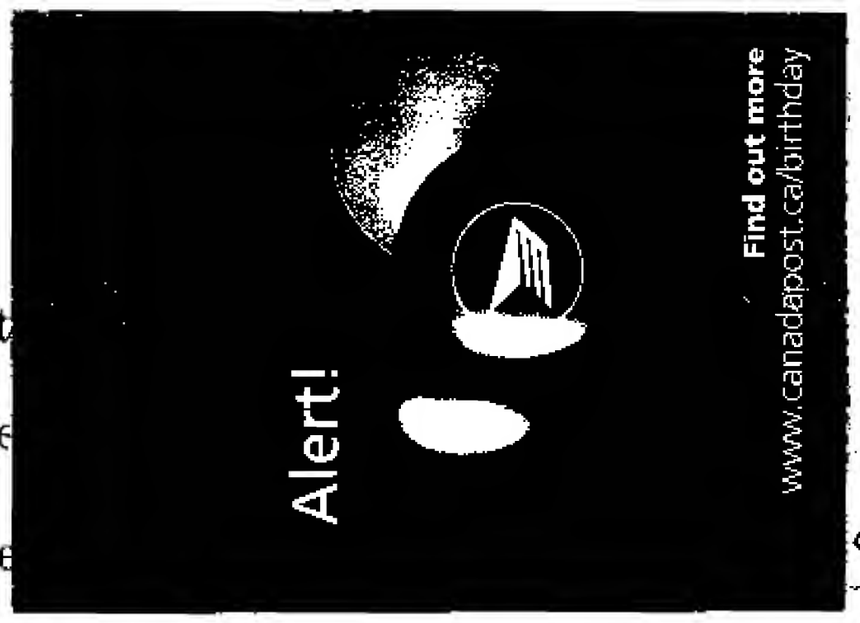


Math 110 Sect

Midterm 2
November 02, 2004
Instructor: Charles Cuell

Name

Stude

All solutions are to be presented on the exam paper in the space provided. Each question is worth two (2) marks. A disorganized or messy solution will result in a mark of zero for that question. *There are nine (9) questions in total on five (5) pages.* Time for the exam is **80 minutes**.

(1) Compute the following. 1 mark each.

(a) $\tan(\frac{3\pi}{4})$

$= -1$

(b) $\cos(3\pi)$

$= -1$

(c) $\sec(-\frac{\pi}{6})$

$\frac{2}{\sqrt{3}}$

(d) $\sin(\frac{19\pi}{6})$ $-\frac{\sqrt{3}}{2}$

(2) Find the solution sets for the following. 1 mark each.

(a) $x^2 - 4 > 0$

$(x+2)(x-2) > 0$

$\{(-\infty, -2) \cup (2, \infty)\}$

(b) $\log_2(3 - x) = 1$

$2^1 = 3 - x$

$2 = 3 - x$

$-1 = -x$

$x = 1$

(c) $\sin x = \frac{1}{\sqrt{2}}$ for $x \in [-2\pi, 2\pi]$

(d) $\cos(2x) = 1$ for $x \in [0, 2\pi]$

(3) Compute the following limits. If the limit does not exist, explain why. 2 marks each.

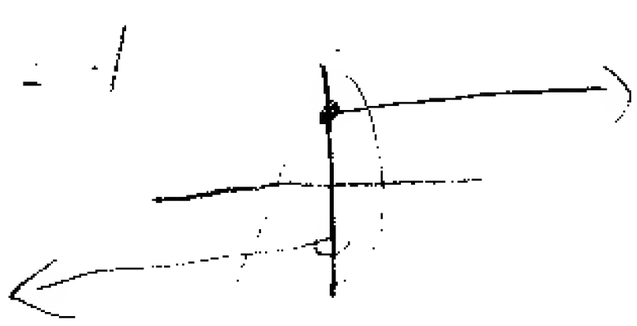
(a) $\lim_{x\to 1^-} \frac{x^2+x-1}{x-1}$

As $x \to 1^-$, $f(x) \to \infty$

(b) $\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x+1}$

(c) $\lim_{x\to 0^-} \log_4(-x)$

(d) $\lim_{x\to 0} \frac{x}{|x|}$ There is No Lim as $x \to 0$



(4) Use the limit definition of a derivative to find $f'(x)$ when $f(x) = \frac{1}{2-x}$. 4 marks.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2-(x+h)} - \frac{1}{2-x}}{h}$$

$$= \lim_{h\to 0} \frac{h(2-x+h) - h(2-x)}{h}$$

$$= \frac{1}{2-x^2}$$

(5) Compute the derivatives of the following functions. 2 marks each.

(a) $f(x) = 2x^3 - 6x + 1$

$$= 6x^2 - 6 + 0$$

$$= 6x^2 - 6 = 6(x^2 - 1)$$

(b) $f(x) = \frac{x^2+1}{e^x}$

$$\frac{2x(e^x) - e^x(x^2+1)}{(e^x)^2}$$

$$= \frac{2x - (x^2+1)}{e^x}$$

(c) $f(x) = (x^8 + 4x^2 - 1)(e^x)$

$(8x^7 + 8x)e^x + (x^8 + 4x^2 - 1)(e^x) = e^x(x^8 + 8x^7 + 4x^2 + 8x - 1)$

(6) Prove that $\frac{d}{dx}(cx) = c$, c a constant directly from the limit definition of the derivative. 4 marks and a deep sense of satisfaction.

$$\lim_{h \to 0} \frac{f((x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{c \quad f(x+h) - f(x)}{h} \quad \frac{(c(x+h) + f(x))}{(c x + h + f(x))}$$

$$\lim_{h \to 0} \frac{c \quad x^2 + h^2 - x^2}{h \quad x + h x} \qquad = c + h$$

$$= c + 0 = c$$

(7) Find the equation of the tangent line to $f(x) = x^2 + 3^x$ at $x = 1$. 4 marks.

$y = 4$

$f'(x) = 2x + (\ln 3)^x$

$m = 2 + \ln 3$

$y - 4 = (2 + \ln 3)(x - 1)$

(8) Let $f(1) = 2$ and $f'(1) = 3$ Evaluate the following. 2 marks each.

(a) $\frac{d}{dx}\left(\frac{f(x)}{x}\right)\Big|_{x=1}$.

$$\frac{f'(x)x - 1(f(x))}{x^2} = \frac{2 - 3}{1}$$

$$= -1$$

(b) $\frac{d}{dx}\left(x^2 f(x)\right)\Big|_{x=1}$

$2(1)(2) + (1)^2 f'(1) = 2 + 3 = 5$

(9) Where is the following function continuous? Analyze any discontinuities and classify as a removeable, jump, or infinite discontinuity. Sketch the graph of this function. 6 marks.

$$f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$$

$\frac{x^2-1}{x-1}$ has a removable discontinuity $\frac{(x+1)(x-1)}{(x-1)}$,

$f(x)$ has a Jump at $x=1$ for a discontinuity.